Assignment - 01

Q. 3 Solve $(x^3+y^3)dx - (x^2y+xy^2)dy = 0$

Solution:- The given diff eqn is a homogenous eqn of degree of 3

$$\frac{dy}{dx} = \frac{(x^3+y^3)}{(x^2y+xy^2)} \quad \text{put } y = vx, \ \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^3+x^3v^3}{x^2vx+xv^2x^2} = \frac{x^3(1+v^3)}{x^3(v+v^2)} = \frac{1+v^3}{v+v^2}$$

$$x\frac{dv}{dx} = \frac{1+v^3}{v+v^2} - v = \frac{1+v^3-v^2-v^3}{(v+v^2)}$$

$$x\frac{dv}{dx} = \frac{1-v^2}{\cancel{v(1+v^2)}\ v+v^2} \quad \text{(By variable seperable method)}$$

$$\frac{\cancel{v(1+v^2)}dx}{\cancel{(1-v^2)}} = \frac{dx}{x} \Rightarrow \cancel{\frac{v(1+v^2)}{(1-v^2)}}dv = \frac{dx}{x}$$

$$\frac{v+v^2}{(1-v^2)}dv = \frac{dx}{x} \Rightarrow \frac{v}{(1-v^2)}dv + \left(\frac{v^2}{1-v^2}\right)dv = \frac{dx}{x}$$

$$\frac{v}{1-v^2}dv + \frac{v^2-1+1}{(1-v^2)}dv = \frac{dx}{x}$$

$$\frac{v}{1-v^2}dv - \frac{(1-v^2)dv}{(1-v^2)} + \frac{1}{(1-v^2)}dv = \frac{dx}{x}$$

integrating both the sides

$$-\frac{1}{2}\log(1-v^2) - v + \frac{1}{2}\log\left(\frac{1+v}{1-v}\right) = \cancel{\log(xc)}\ \log x + \log c$$

$$-\frac{1}{2}\log(1-v^2) - v + \frac{1}{2}\log(1+v) - \frac{1}{2}\log(1-v) = \log x + \log c$$

$$-\frac{1}{2}\log(1-v^2) - v + \frac{1}{2}\log(1+v) - \frac{1}{2}\log(1-v) = \log x + \log c$$

$$-\frac{1}{2}\log((1-v)(1+v)) - v + \frac{1}{2}\log(1+v) - \frac{1}{2}\log(1-v)$$

$$-\frac{1}{2}\log(1-v) - \frac{1}{2}\log(1+v) - v + \frac{1}{2}\log(1+v) - \frac{1}{2}\log(1-v) = \log x + \log c$$

$$-\log(1-v) - v = \log(x) + \log c$$

$$-v = \log(x) + \log(1-v) + \log c$$

$$= \log(x(1-v)) + \log c$$

$$= \log\left(\frac{\cancel{x}(x-y)}{\cancel{x}}\right) + \log c$$

$$-v = \log(c(x-y)) \quad \text{or} \quad \boxed{x-y = e^{-cv}} \quad \text{Ans.}$$

02

04 solve $(3y-7x+7)dx+(7y-3x+3)dy=0$.

Soln The given eqn can be written as

$$\frac{dy}{dx}=\frac{-(3y-7x+7)}{(7y-3x+3)}=\left(\frac{7x-3y-7}{-3x+7y+3}\right)$$

The equation of the form

$$\frac{dy}{dx}=\frac{ax+by+c}{a'x+b'y+c'}$$, Here $a=7,\ b=-3,\ c=-7$ and $a'=-3,\ b'=7,\ c'=3$

$$\frac{a}{a'}\neq\frac{b}{b'}\Rightarrow\frac{7}{3}\neq\frac{-3}{7}$$

putting $x=X+h$, $y=Y+k$ where h and k being const.

$dx=dX$, $dy=dY$

$$\frac{dY}{dX}=\frac{7(X+h)-3(Y+k)-7}{-3(X+h)+7(Y+k)+3}$$

$$\frac{dY}{dX}=\frac{7X-3Y+7h-3k-7}{-3X+7Y-3h+7k+3}$$

Choose the constant in such way such that eqn is being homogenous.

$7h-3k-7=0$

$-3h+7k+3=0$

after solving eqn $h=1$ and $k=0$

$$\frac{dY}{dX}=\frac{7X-3Y}{-3X+7Y}$$ it being homogenous eqn

Put $Y=vX$ $\quad \frac{dY}{dX}=v+X\cdot\frac{dv}{dX}$

$$v+X\cdot\frac{dv}{dX}=\frac{7X-3vX}{-3X+7vX}=\frac{7-3v}{7v-3}$$

$$X\cdot\frac{dv}{dX}=\frac{7-3v}{7v-3}-v$$

$$X\cdot\frac{dv}{dX}=\frac{7-3v-7v^2+3v}{7v-3}$$

$$X\cdot\frac{dv}{dX}=\frac{7\cdot(1-v^2)}{(7v-3)}$$

$$\frac{7v-3}{7(1-v^2)}dv=\frac{dX}{X}\Rightarrow\frac{7v}{7(1-v^2)}dv-\frac{3}{(1-v^2)}dv=\frac{dX}{X}$$

integrating both the sides

$$\int\frac{v}{(1-v^2)}dv-\int\frac{3}{1-v^2}dv=\int\frac{dX}{X}$$

$$-\frac{1}{2}\log_e(1-v^2) - \frac{3}{2}\log_e\left(\frac{1+v}{1-v}\right) = \log_e x + \log_e c$$

$$-\frac{1}{2}\log_e(1-v^2) - \frac{3}{2}\log_e\left(\frac{1+v}{1-v}\right) = \log_e x + \log_e c$$

$$-\frac{1}{2}\log_e(1-v) - \frac{1}{2}\log(1+v) - \frac{3}{2}\log_e(1+v) + \frac{3}{2}\log(1-v) = \log_e x + \log_e c$$

$$\log_e(1-v) - 2\log(1+v) = \log_e x + \log_e c$$

$$\log\left(\frac{1-v}{(1+v)^2}\right) = \log(x\cdot c)$$

$$\frac{1-\frac{y}{x}}{\left(1+\frac{y}{x}\right)^2} = x\cdot c \Rightarrow \frac{x(x-y)\cancel{x}}{\cancel{x}(x+y)^2} = x\cdot c$$

$$\frac{(\cancel{x-1})[(x-1)-y]}{[(x-1)+y]^2} = (\cancel{x-1})\cdot c$$

$$\frac{(x-1-y)}{(x+y-1)^2} = c \quad \text{or} \quad \boxed{(x-y-1) = (x+y-1)^2 c}$$

which is required solⁿ

(05) Solve $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx + (x+\log_e x - x\sin y)dy = 0$

Solⁿ:- Here $M = y\left(1+\frac{1}{x}\right)+\cos y$ and $N = x+\log_e x - x\sin y$

Partially diff M w.r.t y and partially ~~w~~ N w.r.t x

$$\frac{\partial M}{\partial y} = \left(1+\frac{1}{x}\right) - \sin y \quad \text{and} \quad \frac{\partial N}{\partial x} = 1+\frac{1}{x} - \sin y$$

Hence $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ The given Eqⁿ is an exact diff. Eqⁿ. Hence solⁿ is

$$\int_{y=c} M\,dx + \int (\text{the terms of N not containing } x)\,dy = C$$

$$\int \left(y\left(1+\frac{1}{x}\right) + \cos y\right) dx = C$$

$$\boxed{y(x+\log x) + x\cos y = C}$$

which is required solⁿ.

Q.07 solve $(x^3-2y^2)dx + 2xy\,dy = 0$ —— (1)

Soln: Here $M = x^3-2y^2$ and $N = 2xy$

$$\frac{\partial M}{\partial y} = -4y, \quad \frac{\partial N}{\partial x} = \cancel{-2xy} + 2y.$$

~~Th~~ Here $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ Equation is ~~a~~ nonexact

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-4y-2y}{2xy} = \frac{-6y}{2xy} = \frac{-3}{x}$$ it being function of x. Hence integrating factor is

$$I.F = e^{\int \frac{-3}{x}dx} = e^{-3\int \frac{dx}{x}} = e^{-3\log_e x} = \frac{1}{x^3}$$

Equation (1) is multiplied by $\frac{1}{x^3}$

$$(x^3-2y^2)\frac{1}{x^3}dx + \frac{2xy}{x^3}dy = 0$$

$$\left(1-\frac{2y^2}{x^3}\right)dx + \frac{2y}{x^2}dy = 0 \text{ —— (2)}$$

it being exact eqn $\frac{\partial M'}{\partial y} = \frac{-4y}{x^3}, \quad \frac{\partial N'}{\partial x} = \frac{-4y}{x^3}$

where $M' = 1-\frac{2y^2}{x^3}$ and $N' = \frac{2y}{x^3}$

Hence soln

$$\int_{y=c} M\,dx + \int (\text{the terms of } N \text{ not containing } x)\,d\cancel{x}y = c$$

$$\int\left(1-\frac{2y^2}{x^3}\right)dx = c$$

$$x + \frac{2y^2}{2x^2}\cancel{dx} = c \Rightarrow \boxed{x + \left(\frac{y}{x}\right)^2 = c}$$ Required soln.

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(08) solve $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$ —— (1)

Here $M = y^4+2y$, $N = xy^3+2y^4-4x$

$$\frac{\partial M}{\partial y} = 4y^3+2 \quad \text{and} \quad \frac{\partial N}{\partial x} = y^3-4$$

~~∂N~~ Here $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ equation (1) is nonexact

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{y^3-4-4y^3-2}{y(y^3+2)} = \frac{-6-3y^3}{y(y^3+2)} = \frac{-3\cancel{(y^3+2)}}{y\cancel{(y^3+2)}} = \frac{-3}{y}.$$

$$\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M} = \frac{-3}{y}$$ it being function of y

Hence I.F $= e^{\int \frac{-3}{y}dy} = e^{-3\int \frac{dy}{y}} = \frac{1}{y^3}$

Hence eqn ① is multiplied by $\frac{1}{y^3}$

$$(y^4+2y)\frac{1}{y^3}dx + (xy^3+2y^4-4x)\frac{1}{y^3}dy = 0$$

$$\left(y+\frac{2}{y^2}\right)dx + \left(x+2y-\frac{4x}{y^3}\right)dy = 0 \quad \text{②}$$

Here $M' = y+\frac{2}{y^2}$ and $N' = x+2y-\frac{4x}{y^3}$

$$\frac{\partial M'}{\partial y} = 1-\frac{4}{y^3} \quad \text{and} \quad \frac{\partial N'}{\partial x} = 1-\frac{4}{y^3}$$

Equation ② being exact therefor solⁿ of eqⁿ ② is

$$\int_{y=c} M'dx + \int (\text{the terms of } N' \text{ not containg } x)\,dx = C$$

$$\int (y+2/y^2)\,dx + \int 2y\,dy = c$$

$$yx + \frac{2x}{y^2} + \frac{2y^2}{2} = c$$

$$\boxed{x\left(y+\frac{2}{y^2}\right)+y^2 = c}$$ which is required solⁿ

(09) Find an integrating factor in the form $x^m y^n$ then solve $(2x^2y^2+y)dx - (x^3y-3x)dy = 0$

solⁿ The equation can be written as.

$$2x^2y^2dx + y\,dx - x^3y\,dy + 3x\,dy = 0$$

$$(y\,dx + 3x\,dy) + x^2y(2y\,dx - x\,dy) = 0.$$

it being in the following form

$$x^a y^b(my\,dx + nx\,dy) + x^{a'}y^{b'}(m'y\,dx + n'x\,dy) = 0$$

an integrating factor is $x^h y^k$ where

$$\frac{a+h+1}{m} = \frac{b+k+1}{n}, \quad \frac{a'+h+1}{m'} = \frac{b'+k+1}{n'}$$

Here $a=b=0,\ m=1,\ n=3$, $\quad a'=2,\ b'=1,\ m'=2\ \ n'=+1$

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$$\frac{0+h+1}{1} = \frac{k+1}{3} \Rightarrow 3h+3 = k+1 \qquad 3h-k+2=0 \quad (1)$$

$$\frac{2+h+1}{2} = \frac{k+2}{+1} \Rightarrow \frac{h+3}{2} = \frac{k+2}{+1}$$

$$\cancel{-h-3-2k-4=0 \qquad -h-2k-7=0}$$

$$\cancel{h+2k+7=0}$$
$$\cancel{3h-k+2=0}$$

$$h+3 = 2k+4$$
$$\boxed{h-2k-1=0}$$

$$3h-k+2=0$$
$$h-2k-1=0$$

or

$$3h-k+2=0$$
$$3h-6k-3=0$$
$$-\quad+\quad+$$
$$5k+5=0, \quad \boxed{k=-1}$$
$$\boxed{h=-1}$$

Hence $\boxed{I.F = \frac{1}{xy}}$

Multiplying Eqn by $\frac{1}{xy}$

$$(2x^2y^2+y)\frac{1}{xy}dx - \frac{1}{xy}(x^3y-3x)dy=0$$

$$\cancel{\left(\frac{2}{xy}+\frac{1}{x}\right)dx - \left(x^2y-\frac{3}{y}\right)dy=0}$$

$$\left(2xy+\frac{1}{x}\right)dx - \left(x^2-\frac{3}{y}\right)dy=0.$$

and soln is

$$\int\left(2xy+\frac{1}{x}\right)dx + 3\int\frac{dy}{y} = c$$

$$\frac{\cancel{2}x^2}{\cancel{2}}y + \log x + 3\log y = c$$

$$\boxed{x^2y + \log(xy^3) = c}$$ which is required soln.